

A Small Discovery: Avicenna's Theory of *Minima Naturalia*

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ABSTRACT There has been a long-held misconception among historians of philosophy and science that apart from brief comments in Aristotle and Averroes, the theory of *minima naturalia* had to await Latin Schoolmen for its full articulation. Recently scholars have shown that far from sporadic comments on *minima naturalia*, Averroes in fact had a fully developed and well-integrated theory of them. In this study, I complement these scholars' important work by considering Avicenna's place in the history and development of the doctrine of the *minima naturalia*. There is no study to date that mentions Avicenna in connection with this doctrine despite the fact that he dedicated an entire chapter to it in his *Physics*, yet Avicenna's account is at least as developed as and even better integrated than Averroes's presentation. The present study situates Avicenna's position within the more general history of atomism, and introduces Avicenna's "new argument" for natural minima. The argument is important not only for its novelty but also because it shows how Avicenna integrated Aristotle's account of *minima naturalia* into a theory of mixture as well.

KEYWORD Avicenna/Ibn Sīnā, Minima naturalia, Atomism, Continuity/continuum, Mixture

THE THEORY OF *MINIMA NATURALIA* IS LOOSELY one of atoms;* however, unlike the atoms of Democritus, the minimal parts of Epicurus, or the indivisible substances of the *Mutakallimūn*, all of which are physically *and* conceptually indivisible, *minima naturalia*, while being physically indivisible, can be divided at least conceptually *ad infinitum*.¹ Until recently, historians of science believed that with the exception of some passing remarks in Aristotle and some brief exegesis by Greek commentators in the ancient world and then Averroes in the Islamic world, the details of a theory of *minima naturalia* had to await the Latin Schoolmen of the thirteenth century.

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¹While there is a debate whether Democritean atoms should be considered both physically and conceptually indivisible, there is no question about whether the Epicurean and *kalām* sorts are both. For a discussion of the debate concerning Democritus, see Sorabji, *Time, Creation, and the Continuum*, 354–57.

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Ruth Glasner in two groundbreaking works has shown that, far from having just sketchy and sporadic comments about *minima naturalia*, Averroes had a fully developed and well-integrated theory of them.² Unlike earlier historians of science, whose understanding of Averroes's physical theory has been limited to the Latin translation of his great commentary on Aristotle's *Physics*, Glasner consulted all three of Averroes's *Physics* commentaries—short, middle, and long—which requires an appreciation of not only Latin, but also Arabic and Hebrew. As a result of Glasner's careful studies, and even more recently that of Cristina Cerami, we now have a more complete picture of both Averroes's understanding of the *minima naturalia* and his place in the history of atomism.

In this study, I want to complement this important work by considering Avicenna's place in the history and development of the doctrine of the *minima naturalia*. There is no study to date that mentions Avicenna in connection with this doctrine despite the fact that he dedicated an entire chapter to the subject in his *Physics*.³ This omission is no doubt due in large part to the fact that the section where Avicenna discusses this issue was not translated into Latin. Yet Avicenna's account is at least as developed and perhaps even better integrated than Averroes's presentation. (I say 'better integrated' because Avicenna was not limited by the commentary genre that Averroes favored, and so had more freedom to bring things together when and where he saw fit.) In order to situate and to appreciate Avicenna's contribution, then, I begin with a very brief historiography, followed by a short history of *minima naturalia*, starting with the *locus classicus* in Aristotle's *Physics* and going through Aristotle's Greek commentators, with a particular emphasis on John Philoponus. The majority of this study is a presentation of the theory and philosophy behind Avicenna's understanding of the *minima naturalia* that considers what he took from his predecessors as well as what is novel in his theory. I hope to show that despite the neglect that Avicenna has received concerning his place in the history of atomism, he was in fact a pioneer in this subject and actually laid the groundwork from which Averroes, and indirectly subsequent Latin Schoolmen, developed their accounts of a natural minimum.

I. HISTORIOGRAPHY

Historians of science have not represented the contributions of natural philosophers working on the notion of *minima naturalia* in Arabic adequately.⁴ Pierre Duhem in his grand *Le Système du Monde* has only one sentence in which he mentions Averroes's supposed lack of interest in the topic,⁵ and the usually thorough Anneliese Maier wholly neglects the contribution of Muslim thinkers

²Glasner, "Ibn Rushd's theory of *minima naturalia*," and *Averroes' Physics*.

³Avicenna, *Physics*, III.12; the numeric references to Avicenna's works are book number (in Roman numerals), followed by a period and then chapter number (in Arabic). In those cases where this number is followed by a comma and then another Arabic numeral, the second number refers to the paragraph number in the editions of Avicenna's works from The Islamic Translation Series.

⁴For a brief study of the historiography of *minima naturalia*, see Murdoch, "The Medieval and Renaissance Tradition of *Minima Naturalia*," 91–131, esp., 91–96.

⁵*Ce texte ne semble guère avoir retenu l'attention d'Averroès, car celui-ci se borne à écrire, en son commentaire: "il est manifeste de soi que le volume de la chair est limité en grandeur comme en petitesse"* (Duhem, *Le système du monde*, VII.42).

on this topic.⁶ Andrew G. van Melsen dedicated approximately two pages of his study on atomism, *From Atomos to Atom*, to Averroes's account available in Latin.⁷ Norma Emerton likewise reserved about as much space in her work *The Scientific Reinterpretation of Form* to Averroes's theory of *minima naturalia*, but now with an eye to how that theory was linked with another important medieval debate—namely, the theory of *mixtion* or primary mixture.⁸

Ruth Glasner is the first to begin giving philosophers working in the medieval Islamic world their due on this subject. In both an article and chapter of her book, *Averroes' Physics*, she chronicles Averroes's place in the history of the *minima naturalia*. Ostensibly, Averroes's account of the *minima naturalia* was, Glasner tells us, an attempt to reconcile seemingly contradictory claims in Aristotle's *Physics*. The first such claim is Aristotle's criticism of Democritean atomism, which committed Aristotle to the belief that all natural magnitudes are continuous and so potentially divisible infinitely. The second is Aristotle's critique of Anaxagoras, in which he maintains that there is a limit beyond which natural substances cannot be further divided.⁹ Even more recently Cristina Cerami, while challenging some of Glasner's claims, has further extended our knowledge of Averroes's theory of the *minima naturalia*.¹⁰ In none of the works of this set of historians, starting with Duhem and going through Glasner and Cerami, is there ever mention of Avicenna's place in this history. Before I can tell his story, however, I need to begin by relating the origins of the idea of *minima naturalia* in, first, Aristotle, and then some of his later Greek commentators, most notably John Philoponus.

2. A HISTORY OF MINIMA NATURALIA IN THE GREEK WORLD

Aristotle introduces the idea of natural minima (*elachista*) at *Physics* 1.4, which became the *locus classicus* for the doctrine of *minima naturalia*. In that passage, he critiques Anaxagoras's principle that "everything is in everything."¹¹ Arguably, Aristotle's most important proof concerning *minima naturalia* runs thus:

If the part can be indefinitely big or small—I call a 'part' the ingredient into which the whole is divided—then necessarily the thing itself can be too. So if it is impossible that an animal or plant be indefinitely big or small, then clearly the part is not such as [to be indefinitely big or small] either, for the whole also will be such. Now, flesh, bone, and the like are parts of an animal, and fruits [the parts] of a plant. Hence, it is clear that flesh, bone, and the rest cannot be indefinitely big in the direction of greater or smaller. (*Physics* 41.4, 187b13–21)¹²

Aristotle's thesis is that the parts that constitute natural kinds—parts such as flesh, blood, bone, fruits, and the like—have a definite limit with respect to their size,

⁶Maier, *Die Vorläufer Galileis im 14. Jahrhundert*, 179–96.

⁷Van Melsen, *From Atomos to Atom*, 58–60.

⁸Emerton, *The Scientific Reinterpretation of Form*, 87–88.

⁹Glasner, "Ibn Rushd's theory of *minima naturalia*," 10–14.

¹⁰Cerami, "Mélange."

¹¹For a discussion of Anaxagoras's principles, see Drozdek, "Anaxagoras and the Everything in Everything Principle."

¹²All translation, whether Greek or Arabic, are my own.

both in greatness and smallness, namely, a minimum and maximum. The argument for this thesis—limiting the present discussion merely to the idea of *minima*—is straightforward enough. Aristotle takes it as impossible that cats, for example, should be indefinitely small. Thus, any premise that leads to the possibility of indefinitely small cats must be false. If Anaxagoras is correct, and everything is in everything such that it is possible, at least in principle, to extract out from a given substance indefinitely small cat flesh, cat blood, and all the sundry cat organs and bits, then these indefinitely small cat parts should, again in principle, be able to constitute an indefinitely small cat. Since the assumption is that indefinitely small cats are impossible, the premise that gave rise to the absurdity, namely, that there are indefinitely small traces of everything in everything, Aristotle concludes, must be false. Later in the same passage, the existence of *minima naturalia* plays an essential role in two other arguments against Anaxagoras, but in those passages Aristotle takes the existence of *minima naturalia* as demonstrated, presumably on the strength of the argument that I have just presented.¹³

According to the medieval Arabic bibliographers, al-Nadīm and al-Qifṭī, commentaries on book I of the *Physics* by the later Greek commentators, Alexander of Aphrodisias, Themistius, and John Philoponus were available in Arabic translation. Thus, in addition to Aristotle's own *Physics*, these are likely the Greek works to have influenced the understanding concerning *minima naturalia* of subsequent natural philosophers working within the medieval Islamic milieu. Of these, Alexander's commentary is no longer extant in either Greek or Arabic, with Simplicius preserving only fragments of it in his own *Physics* commentary. Additionally, there are a few scattered fragments of Alexander's commentary extant in Arabic but none from book I.¹⁴ As for Themistius's *Paraphrase*, it remains fairly close to Aristotle's text. In contrast, Philoponus's commentary exists both in Greek and Arabic and is somewhat extensive.¹⁵

In addition to the *Physics* 1.4 passage, which is Aristotle's clearest statement concerning *minima naturalia*, scholars also see the doctrine hiding behind Aristotle's comments in *On Generation and Corruption* 1.10 and in *On Sense and Sensibilia* 6. *On Generation and Corruption* 1.10 includes Aristotle's theory of primary mixture—what historians of science sometimes refer to as *mixtion*. As will become apparent, this passage seemed to exert some influence on Avicenna's theory of *minima naturalia*. As for the fate of *On Generation and Corruption* in the Arabic world, unlike the *Physics*, the Arabic translation of that text is no longer extant. In addition to the work itself, al-Nadīm and al-Qifṭī mention the commentaries of, again, Alexander of Aphrodisias, Themistius, and John Philoponus. Of these, Themistius's commentary is not extant. Alexander's commentary was thought to be lost, but an Arabic translation of *On Generation and Corruption* 2.2–5 was preserved in the alchemical work, *Kitāb al-Taṣwīf* (*Book of Transformation*), of Jābir ibn Ḥayyān (721–815), but again it would seem nothing concerning our passage remains.¹⁶

¹³See Aristotle, *Physics* 187b27–32 and 187b35–188a2.

¹⁴See Giannakis, "Fragments from Alexander's Lost Commentary on Aristotle's *Physics*."

¹⁵Philoponus (=Yāhyā), *In Phys.*

¹⁶See Gannagé, "Le commentaire d'Alexandre d'Aphrodise *In de generatione et corruptione*," and *On Aristotle's On Coming-to-be and Perishing* 2.2–5.

Finally, while the Arabic translation of Philoponus's commentary of *On Generation and Corruption* is no longer extant, the complete text still exists in Greek.¹⁷ As for *On Sense and Sensibilia*, al-Nadīm claims that the work was unknown even at the end of the tenth century. Still, there appears to be evidence that Avicenna had access to some form of it, although whether this text influenced his theory of *minima naturalia* is perhaps impossible to say.¹⁸

In light of the foregoing, Philoponus's discussions of the relevant passages are clearly the most complete and latest of the Greek commentaries, and thus incorporate many of the earlier advances on the topic of the *minima naturalia*. Moreover, Philoponus's commentary (certainly his *Physics* commentary) had the greatest apparent influence on Avicenna's understanding of Aristotelian physics. Hence, I focus primarily upon Philoponus's account of natural minima.

The most notable advancement in Philoponus's commentary is that he shifts the discussion from natural substances and their parts to the form (*eidos*) of those substances and their parts. This shift does not appear to be new to Philoponus. Themistius occasionally mentions form in his exposition of our passage.¹⁹ In fact, the introduction of form may go back as far as Alexander of Aphrodisias, since Simplicius suggests that Alexander and Themistius interpreted Aristotle similarly.²⁰ The introduction of form is probably part of the commentators' larger project of developing an integrated Aristotelian physics. So, for example, in his *On Generation and Corruption* 1.10 commentary, Philoponus situates Aristotle's discussion of mixture within the broader context of refuting Anaxagoras's principle, "Everything is in everything," and Philoponus's theory of mixture developed there presupposes the account of form presented in arguing for *minima naturalia*, a point to which I return below.²¹

Turning to Philoponus's version of the argument for *minima naturalia*, he begins by claiming, "Every form naturally subsists in some definite quantity, and it is neither naturally augmented to just any degree of largeness nor naturally diminished to just any degree of smallness, but rather there is a certain boundary to greater and smaller beyond which the form does not exist."²²

Philoponus next attempts, in perhaps a not altogether successful way, to defend his newly modified major premise, namely, that the form cannot exist beyond a certain natural maximal and minimal quantity. This defense itself goes beyond Aristotle, for Aristotle merely asserted that natural substances could not be of

¹⁷Philoponus, *In Gen. et Corr.*

¹⁸See Peters, *Aristoteles Arabus*, s.v. "Liber Sensus et Sensati," 45–47.

¹⁹Themistius, *In Aristotelis Physica Paraphrasis*, 14, 25–26 and 15, 13–16.

²⁰Simplicius, *In Aristotelis Physicorum Libros Quattuor Priores Commentaria*, 169, 5–25. Unfortunately, while Simplicius quotes Alexander extensively, there is no mention of Alexander's using form in his interpretation. Still, such an absence need not be an indicator that Alexander did not re-frame the argument in terms of form, since Simplicius's emphasis in quoting Alexander is on the apparent fact that Alexander's version of Aristotle's *Physics* differed from that of Simplicius's own.

²¹For the significance of Anaxagoras's principle, see Philoponus, *In De Gen. et Corr.*, 192, 10–16; and the same for the attenuation or reduction of form; and Philoponus, *In De Gen. et Corr.*, 198, 18–19, for form's need of a minimum quantity. Discussions of Philoponus's theory of mixture can be found in De Haas, "Mixture in Philoponus," and Wood and Weisberg, "Interpreting Aristotle on Mixture."

²²Philoponus, *In Phys.*, 96, 27–30.

just any size whatsoever. In the case of a maximum, Philoponus claims that the form fades, or becomes attenuated (*exitēla*), the more it must spread throughout a given quantity.

Indeed no human [for example] would come to be a hundred feet or equal to the size of the cosmos; for we are not assuming some human in thought but in reality. Clearly, then, [a real human] cannot be augmented to just any size and beyond all. Instead, there is a certain limited size beyond which [a real human] cannot be augmented, *for the form extended over a large subject becomes attenuated.* (Philoponus, *In Phys.*, 97, 4–9)

Presumably, then, there are decided maxima beyond which forms simply perish, as the drop of wine perishes as it spreads throughout 10,000 gallons of water.²³ Philoponus next posits that just as there is a maximum, so there must be a minimum quantity beyond which form cannot exist. It should be noted, however, that he merely asserts this last point and does not provide an argument for it; for the attenuation argument that he used to justify a natural maximum clearly is inapplicable to minima, and, if anything, there would be a concentration of the form.

Philoponus next turns to a mathematically motivated objection to the idea of *minima naturalia*. This objection presupposes Aristotle's discussion of whether magnitudes are continuous or discrete from *Physics* 6, and so we should briefly linger over this issue. The driving question of *Physics* 6 is whether magnitudes such as distances, motion, and time are continuous, and so are potentially divisible *ad infinitum*, or whether they are discrete such that a process of division ultimately terminates in certain indivisible parts or atoms. Despite what we have seen in Aristotle's *Physics* 1.4 concerning his belief in *minima naturalia*, Aristotle unequivocally denies that magnitudes are composed of atoms, and instead holds that all magnitudes must be continuous, and so potentially divisible infinitely. Moreover, unlike his passing remarks concerning *minima naturalia*, Aristotle spends all of *Physics* 6 engaged in, at times, highly technical argumentation that magnitudes must be continuous.

Given the seeming discrepancies between *Physics* 1.4 and book 6, the objection that Philoponus considers is a pressing one. It runs thus:²⁴ since all magnitudes are continuous and so potentially divisible *ad infinitum*, let the purported minimum amount of flesh, for example, be divided. The resultants of the division are either themselves flesh or they are not flesh. If they are flesh, then there are quantities of flesh less than the minimum amount of flesh, which is absurd. "If the divided things are not flesh, then how will they produce the composite flesh again? If the flesh is homoeomerous, clearly the parts of this would be flesh too."²⁵ This last horn of the dilemma, to which I return again when considering Avicenna's account of *minima naturalia*, can be framed thus: If the resultants of the division are not flesh, then a composite of flesh would not be uniformly flesh through and

²³Philoponus does not make this final point here, but the example and the assertion that forms require a certain quantity are given in Philoponus, *In De Gen. et Corr.*, 198, 11–19.

²⁴Philoponus, *In Phys.*, 98, 13–21.

²⁵Philoponus, *In Phys.*, 98, 17–19.

through, but instead would be a collection of distinct non-flesh parts, which the objector and Philoponus find absurd.²⁶

Philoponus's response to this objection again draws on his introduction of form to explain *minima naturalia*, for, says Philoponus, flesh can be considered either qua form or qua magnitude. Qua magnitude the flesh is continuous and so potentially divisible infinitely, in which case there is no minimal magnitude.²⁷ This is all that Philoponus says here about division qua magnitude and unfortunately he leaves the notion of division (*diairesis*) underdetermined.²⁸ Division here could mean physically dividing the magnitude into smaller and smaller portions or merely conceptually dividing the magnitude, as in the mathematical series 1, 1/2, 1/4, 1/8, 1/16, . . . 1/2ⁿ . . . ∞. Since the mathematicians (*hoi apo tōn mathmētōn*) raise the puzzle, one might suspect that the latter form of division is meant, and yet when Philoponus turns to division qua form, it would seem that he intends physical division. I return to the suggestion that there are two distinct kinds of division when discussing Avicenna on division.

As for division qua form, the flesh qua form is not infinitely divisible. Any division that results in a quantity less than that required for the subsistence of the form brings about the destruction of the form. To make his point graphically, he has us take as an example a human. On the one hand, we can consider the human qua magnitude as, for instance, 6 feet tall. In this case the individual can be divided into indefinitely smaller magnitudes: 3 feet, 18 inches, 9 inches, and so on. While there are smaller and smaller magnitudes, magnitude is never completely destroyed but always remains. On the other hand, if we consider the human qua (human) form, then, when we divide our ill-fated victim into feet, legs, torso, and head, we do not have smaller instances of the human form. We simply no longer have the form of human at all but instead a dismembered corpse. Species forms, in short, cannot survive division the way magnitudes can.

Similarly, says Philoponus, if there is to be flesh at all, that flesh requires the presence of the form of flesh, and the form of flesh is dependent upon some minimal quantity of matter. The inference is perhaps a bit too quick, for while it seems fairly obvious that the form of human is not fully localized in any part of the person, like the head, it is not equally clear that the form of flesh is not fully present in any bit of flesh regardless of how small. The problem is that even if one is convinced that Philoponus's arguments work at the level of the species form, it is not as obvious how those arguments translate at the level of the forms of mixtures.

Whatever the limits of Philoponus's argument, he has certainly gone beyond Aristotle and helped motivate Aristotle's original argument. Natural substance must have natural minima, since there are minimum quantities required for the subsistence of forms. As for how Philoponus's version of the argument is an advancement over Aristotle's original version, consider the following. Aristotle

²⁶In fact, Aristotle also commits himself to the position that flesh, blood, and the like are uniformly flesh, blood, etc., through and through, when he develops his theory of primary mixture in *On Generation and Corruption* I.10.

²⁷Philoponus, *In Phys.*, 98, 21–2.

²⁸For a discussion of division within Philoponus, see De Haas, *John Philoponus' New Definition of Prime Matter*, 116–20.

frequently notes that substance (*ousia*) is said in three ways: matter, form, and the composite.²⁹ Moreover, of these different ways of speaking of substance, there is a sense that substance-as-form is primary.³⁰ Aristotle's original argument, however, was solely in terms of natural (i.e. composite) substances. Thus, at the very least, Philoponus's argument in terms of form is a generalization of Aristotle's original argument, recasting it now in terms of the more general or basic notion of substance-as-form.

Still, there is also a sense in which Philoponus's presentation might not be merely commentary but a new argument altogether. As noted above, a key element in Philoponus's version of the argument is that the form fades and becomes attenuated. This conception of a form's being able to fade is not unique to Philoponus's *Physics* commentary but is found in other works by him as well. For example, in commenting *On Generation and Corruption* 1.10, Philoponus explains the difference between generation (*genesis*) and mixture (*mixis/krama*) thus:

Concerning generation, the matter of the air [for example] is potentially air but actually water, while in a mixture what is mixed subsists potentially, not the matter itself but rather the very *form is reduced* [*kekolasmena*]. Because of this it is in potency, since it is neither pure nor such as it was before the mixing. (*In De Gen. et Corr.*, 192, 10–14; my emphasis)

It would seem that Philoponus has some idea of the intension and remission of (species!) forms.³¹ It would go well beyond the scope of the present paper to adjudicate as to whether Philoponus in fact has a doctrine of intension and remission of forms as well as whether the historical Aristotle may have held that theory. Nonetheless, it is safe to say that if Philoponus's theory of form is substantively different from Aristotle on this point (and my suspicion is that the two are different), then the present argument represents a creative moment in the history of atomism.³²

3. THE AVICENNAN BACKGROUND TO THE *MINIMA NATURALIA*

The preceding discussion provides roughly the theory of *minima naturalia* as it appeared at the end of the late Antique Period and as it would have been passed on to the medieval Islamic world. Turning now to Avicenna, his theory of *minima naturalia* comes at the end of book III of his *Physics*, which as a whole is dedicated to the topic of discrete and continuous magnitudes. Consequently, Avicenna's account of *minima naturalia* presupposes two things: an understanding of his rejection of atomism—both as atomism was inherited from the Greek tradition

²⁹Aristotle, *De anima* 2.1, 412a6–9; *Metaphysics* 7(Z).3, 1028b33–1029a3; and *Metaphysics* 8(H).1, 1042a24–31.

³⁰See, for instance, *Metaphysics*, 7(Z).3, 1029a26–33.

³¹While the later medieval tradition certainly found the doctrine of *insensio et remissio* in the works of Aristotle, it was with respect to accidental forms. For the medieval Latin context see Sylla, "Medieval Concepts of the Latitude of Forms: The Oxford Calculators," and *The Oxford Calculators and the Mathematics of Motion, 1320–1350: Physics and Measurement by Latitudes*; also see Dumont, "Godfrey of Fontaines and the Succession Theory of Forms."

³²One study that touches on Philoponus's theory of forms in a significant way is Macierowski and Hassing, "John Philoponus on Aristotle's Definition of Nature."

and more importantly as contemporary Muslim theologians conceived it—and an appreciation of his defense and (somewhat novel) understanding of the continuum. Thus, let me provide some background to his view concerning discrete and continuous magnitudes.

Perhaps the most important thing to bear in mind when considering Avicenna's critique of atomism is the nature of the atoms that he wants to reject. He happily concedes that there might be bodies for which there are no physical means to divide them further. He thus recognizes that certain substances may as a matter of fact be indivisible, and so in a literal sense be atoms. The philosophically dubious atoms, at least by Avicenna's lights, are those that are not only physically indivisible, but also, and more importantly, conceptually indivisible. These are the minimal parts of Epicurus and the indivisible parts of the Islamic speculative theologians (*al-juz' alladhī lā yatajazza'u*).³³ Despite the fact that these atoms were believed to be conceptually indivisible, they were nonetheless thought to be space occupying (*mutaḥayīz*). For many it might seem problematic to predicate of atoms both that they occupy space—and so, it would seem, are extended—and yet that they are conceptually indivisible. Be that as it may, there were a number of puzzles, involving purported absurdities surrounding continua and their infinite divisibility, that were quickly resolved if one had an ontology of discrete, that is, atomic, magnitudes.

As for the actual arguments that Avicenna uses to dismiss atomism and embrace continua, these need not bother us here.³⁴ Suffice it to say that Avicenna believed that any purportedly indivisible magnitude could be divided into conceptually distinct parts. Moreover, he argued that without positing continua, the conclusions of Euclidean geometry could not even be approximated, and yet geometry was the most well established science of the time. Finally, as for the absurdities that seemingly arise from the infinite divisibility of continua, Avicenna maintained that once one properly understands the nature of the continuum, those absurdities are seen for the sophistries that they are. (I return to this final point at the end of this section.)

Thus, turning to Avicenna's theory of the continuum, a complete account would require a discussion of his theory of matter, corporeality—or more exactly, the form of corporeality (*ṣūrat al-jismīya*)—and three-dimensionality, most of which would take me far afield from the issue of *minima naturalia*.³⁵ Instead, allow me to focus primarily on Avicenna's fullest treatment of the continuum as it appears at *Physics* III.2, with the occasional side remark about the other issues to clarify his discussion.³⁶ While *Physics* III.2 is Avicenna's most detailed discussion of

³³For Epicurus, see Furley, *Two Studies in the Greek Atomists*, and for the *mutakallimūn*, see Dhanani, *The Physical Theory of Kalām*, 90–140. For Avicenna's knowledge of both Greek and Islamic atomism, see Avicenna, *Physics*, III.3.

³⁴See Avicenna, *Physics*, III.4. For a study of Avicenna's refutation of atomism, see Lettinck, "Ibn Sīnā on Atomism."

³⁵For discussions of the relation between matter and corporeality, see Avicenna, *Physics*, I.2, 4, and *Metaphysics*, II.2–3. For studies see Hyman, "Aristotle's 'First Matter' and Avicenna's and Averroes' 'Corporeal Form,'" and Stone, "Simplicius and Avicenna on the Essential Corporeity of Material Substance."

³⁶For a discussion of Aristotle's various treatments of the continuous, which influenced both positively and negatively Avicenna's own understanding, see Glasner, "Ibn Rushd's Theory of *Minima Naturalia*," 11.

continuity, taken alone it can be disjointed in places. Fortunately, certain remarks that he makes about the continuum in a letter to the Vizier Abū Saʿd significantly smooth out the *Physics* discussion.³⁷ Both texts observe that continuity (*ittiṣāl*) is an equivocal notion, and in the *Physics*, Avicenna identifies three ways in which one might speak of the continuous.³⁸ Two senses of ‘continuous’ are understood relative to something else (Avicenna mentions only one of these senses in his letter to Abū Saʿd), while the third concerns the continuous considered in itself.

One of the relative senses of being continuous identified in the *Physics*—the one omitted in the letter to Abū Saʿd—is said of an object inasmuch as it is moving. This form of continuity occurs, Avicenna tells us, when:

. . . one side of the continuous thing is moved in a direction away from the other, the other follows it. . . . The two extremities can be two in actuality, and there can be something actually contiguous after adhering during the motion. The extremity of what is continuous and that with which it is continuous can be one, but it is termed *continuous* in the present sense not inasmuch as its extremity and that of the other are one, but only inasmuch as it follows it during the motion in the aforementioned way. (Avicenna, *Physics*, III.2, 9)

Here continuity is relative to the motion. Thus, for example, if one considers a train, the engine, caboose, as well as any intervening boxcars do not share a common limit, and yet they move together such that relative to that motion one can say of them that they make up a continuous moving whole.

The second relative sense, which Avicenna considers, in both his *Physics* and the letter to Abū Saʿd, is the continuous relative to a limit (*taraf*), which he finds in Aristotle’s *Physics* 5.3 227a11–12, namely, “that whose limit is the same as a limit of something else.” This case occurs when something is continuous relative to a shared limit that is one and the same for two parts. (In his own *Physics*, Avicenna unfortunately gives the impression that there are two subspecies of this relative notion of continuity: absolute and accidental. In the letter to the Vizier Abū Saʿd, however, it is evident that the accidental continuity that Avicenna discusses in the *Physics* is in fact the continuous in itself.)³⁹ An example of this second relative sense of being continuous is two lines forming an angle, since each line has one and the same common point at the angle’s vertex, and so can be said to be continuous relative to the limit according to Aristotle’s *Physics* 5.3 account of continuity. Nonetheless, the two parts are actually distinct even though they share one and the same common limit.

Avicenna’s account of the continuous in itself, which he perhaps misleadingly also labels as accidental continuity, is he tells us, “the definition that is mentioned in [Aristotle’s] *Categories* [6, 5a1–2], namely, ‘that for whose parts a common limit can be found at which they meet.’”⁴⁰ In his *Physics*, Avicenna describes the continuous in itself as that magnitude that in itself has no parts (*lā juzʿ*),⁴¹ but in which one

³⁷For the text and a French translation, see Avicenna, *Letter to Abū Saʿd*.

³⁸Avicenna, *Physics*, III.2, 8–10.

³⁹Avicenna, *Letter to Abū Saʿd*, 42–44.

⁴⁰Avicenna, *Letter to Abū Saʿd*, 43.

⁴¹The description of the continuous in itself having no parts is explicitly made at Avicenna, *Physics*, III.3, 1.

can posit limits in it in an accidental sense, a point to which I return shortly. The most obvious instance of parts that the continuous in itself must lack is that of physical parts, as in the case of, for example, the articulation of the bones of the arm, and, more generally, actual parts, as in the two sides of the angle mentioned above. Additionally, the continuous should not be thought to have even latent parts present within it waiting to emerge into actuality. This theory of latency (*kumūn*) is most commonly associated with Ibrahim al-Nazzām (c. 775–845). Avicenna clearly wants to distinguish his theory of continuity from al-Nazzām's, who believed that an actual infinity of parts are latent within a continuous object.⁴²

For Avicenna, the continuous only ever has accidental parts, parts that result through a psychological act of positing accidental limits within the continuous object. Avicenna describes these accidental limits thus:

[They are] like what happens when our estimative faculty imagines or we posit two parts for a line that is actually one, where we distinguish one [part] from the other by positing. In that way, a limit is distinguished for [the line] that is the same as the limit of the other division. In that case, both are said to be continuous with each other. Each one, however, exists individually only as long as there is the positing, and so, when the positing ceases, there is no longer *this* and *that* [part]; rather, there is the unified whole that actually has no division within it. Now, if what occurs through positing were to be something [really] existing in the thing itself and not [merely] by positing, then it would be possible for an actually infinite number of parts to exist within the body (as we shall explain), but this is absurd. (*Physics*, III.2, 8)

Accidental limits thus occur within the continuous in itself when the single unified continuous whole is distinguished into two (or more) conceptual parts through some act of positing (*farḍ*), such as pointing toward a uniform surface and saying, “this side,” while pointing to the right, and “that side,” while pointing to the left. The limit in this case—and Avicenna is adamant about this point—arises only as an accidental result of the positing, and in fact that limit ceases once the positing ceases.⁴³ Indeed, to maintain that the part still remains after the pointing stops, says Avicenna, is tantamount to saying that the pointing itself remains when the pointing has stopped. It is simply false, warns Avicenna, to think that the limit in this case really exists in the continuum.

Moreover, he warns his reader not to mistake the description (*rasm*) of the continuous in Aristotle's *De caelo* (I.1, 268a5–6)⁴⁴ which is given in terms of “that which can be divided into things always susceptible to [further] division,” as constituting the essence (*māhīya*) of the continuous in itself.⁴⁵ In other words, it is not the essence of the continuous to have a potential infinity of divisions within it. Instead, this description is at most a concomitant of the continuous, which must be demonstrated to belong to it necessarily. In other words, one must be careful even when one speaks of potential limits inhering within the continuous in itself, if by ‘potential’ one means, again, something latent within the continuum waiting to be actualized.

⁴²For a discussion of al-Nazzām and his theories, see Wolfson, *The Philosophy of Kalām*, ch. II and VI.

⁴³Avicenna reiterates this point in even stronger terms at *Physics*, III.5, 3.

⁴⁴Also see Aristotle, *Physics*, 6.2, 232b24–5.

⁴⁵Avicenna, *Letter to Abū Sa'īd*, 43; and Avicenna, *Physics*, III.2, 8.

Avicenna takes up the issue of a continuous magnitude and its divisibility (*inqisām*) in earnest at *Physics* III.9. Here we learn that “the magnitude . . . in fact is the continuity itself, not some continuous thing resulting from a continuity in [in the magnitude].”⁴⁶ While this claim is part of a technical discussion concerning matter, in general, Avicenna wants to deny that prime matter considered independent of any form has a magnitude of itself, to which the property of being continuous is then superadded.⁴⁷ Instead, for Avicenna, matter is a body owing to the form of corporeality, and the body’s being a continuous magnitude is due to that form of corporeality.

As for the divisibility of the magnitude, he unequivocally identifies two senses in which one speaks of divisibility.

One of them is [in the literal sense of] *discontinuity* and *becoming separated*, which follows upon quantity owing to the preparedness of the matter. The other sense of divisibility is that it is in the nature of the thing that one thing in it is *posited* as different from some other thing and so on without coming to an end, which essentially follows upon magnitude. . . . The first [sense] is true divisibility, namely, what changes the state of the thing, whereas the second is only a function of the estimative faculty [*mawhūm*]. (*Physics*, III.9, 3)

For Avicenna, there is, clearly, “physical divisibility” and “conceptual divisibility.” The first and true sense of divisibility for Avicenna, that is, physical divisibility, is the form of division that actually brings about a separation and discontinuity within the magnitude. It involves physically dividing a magnitude into two actually distinct parts, and in so doing destroying the continuity and so the original magnitude itself.⁴⁸

The second type of division, namely, conceptual divisibility, involves only the accidental partition of the magnitude. The parts involved in this type of divisibility are the accidental parts noted above that result from a certain psychological process, namely, division through mere positing. In the present discussion, Avicenna throws additional light on his conception of the continuous, for we now learn that within the magnitude there is merely a preparedness (*isti d d*) for being divided, and “that preparedness need not belong to a form, for it is not the case that what acts must do so in itself.”⁴⁹ The notion of preparedness or receptivity is one of Avicenna’s preferred descriptions for matter (*hayūlā*). Since for Avicenna matter has no formal or positive feature of its own, the preparedness of the magnitude almost certainly means that there is nothing about the underlying matter that prevents the magnitude’s being divided infinitely. Thus, to the question, “Why is a continuous magnitude infinitely divisible?” the answer is “Because the matter of the continuous magnitude does not prevent its being so divided.” The process of division ceases only if one terminates the dividing process because, for instance, there is no fitting tool, or the one doing the dividing could not go on, or some form *in addition* to the form of corporeality prevents its being further divided.⁵⁰ It

⁴⁶Avicenna, *Physics*, III.9, 3; also see *Physics*, III.2, 8–10, and *al-Maqūlāt*, III.4.

⁴⁷For an overview of the classical context to Avicenna’s claim, see Sorabji, *Matter, Space, & Motion*, I. Matter, esp. sections 1–3; and De Haas, *John Philoponus’ New Definition of Matter*, ch. 2, sect. III.

⁴⁸Avicenna, *Physics*, III.9, 3.

⁴⁹Avicenna, *Physics*, III.9, 4.

⁵⁰Avicenna, *Physics*, III.3, 1.

is not because the body as such is insusceptible to physical divisions as small as one likes. The preparedness of matter, then, plays the role of the middle term in demonstrating the infinite divisibility of a continuum, which we saw him mention at *Physics* III.2.

Moreover, the reference to preparedness is another way to emphasize that there are no real or formal parts within a continuous magnitude; for matter is wholly passive and considered independent of any form that could not bring about any formal features in a thing. Instead, it is again the estimative faculty's positing accidental limits—and so accidental parts within a continuum as small as one likes—that accounts even for the accidental parts within a continuum. In short, a continuum's infinite divisibility refers to a potentially never-ending psychological process, not to some formal feature(s) actually existing within the magnitude itself.⁵¹

Since Avicenna has now introduced the estimative faculty (*wahm*) and its product on multiple occasions to explain the continuous in itself, and the continuum's being described as potentially infinitely divisible, a brief word is warranted about his notion of estimation.⁵² The estimative faculty is one of Avicenna's celebrated internal senses, whose function is to receive the connotational attributes (sing. *ma'nā*) inherent in material objects. A connotational attribute is a non-sensible property or quality of a sensible or material object. The classic example is the sheep's perception of the ferocity of the wolf. For while the sheep can see the long claws and teeth of the wolf as well as smell its scent and hear its growl, none of the external senses perceive the wolf's ferocity itself. Yet in some sense the sheep perceives that ferocity. It is the estimative faculty that perceives this connotational attribute of ferocity. More importantly for our story, the estimative faculty is that power by which imagination (*mutakhayyila*) combines and divides both sensible forms and connotational attributes.⁵³ Now, within his *Physics*, Avicenna's preferred language for perceiving temporal and spatial relations as well as setting up thought experiments that cannot in principle be done—like imagining the infinite divisibility of a continuum—is always that of *wahm* and its cognates. Thus, the role of the estimative faculty in understanding the continuum is literally to imagine parts within a magnitude in which those parts simply are not present in any way whatsoever, save as accidental concomitances of the estimative faculty's act of positing.

Before turning to Avicenna's account proper of *minima naturalia*, let me conclude this section by saying something about Avicenna's adamant, almost paranoid, denial that the continuous in itself has anything like parts or latent divisions.⁵⁴ To help contextualize Avicenna's account of continuity, it must be noted that the *mutakallimūn*, that is, Muslim (speculative) theologians, had viciously attacked the philosophers' notion of the continuum and precisely on the point that the

⁵¹See Avicenna, *Physics*, III.2, 8; III.5, 3; and III.9, 3–4.

⁵²For specialized discussions of Avicenna's theory of the estimative faculty, see Black, "Estimation (*Wahm*) in Avicenna," and Hasse, *Avicenna's De Anima in the Latin West*, II.4, "Estimation and 'Intentions,'" 127–53; and for a general overview see McGinnis, *Avicenna*, 113–16.

⁵³For a discussion of the (compositive) imagination, see Black, "Rational Imagination: Avicenna on the Cogitative Power."

⁵⁴The following is drawn from McGinnis, "Avicenna's Natural Philosophy," §2, 78.

essence of the continuum is its potential infinity divisibility. These *kalām* thinkers simply noted that in order for something to be truly potential, there must be a corresponding power that can actualize that potential, whether it is the finite power of some created thing or the infinite power of God.⁵⁵ Thus, the objection continues, if a continuum were to exist, and if something is continuous if and only if it has a potentially infinite number of divisions inherent in it, then, to the extent that a continuum purportedly has a true potential for infinite divisibility, there must be some power that can actualize the potential infinity of divisions. If that potential is actualized, however, an actual infinity of divisions would be possible, for just let the corresponding power actualize the infinity of potential divisions, in which case there would actually be an infinity of divisions. According to the Aristotelians' own position, however, an actual infinity is impossible. What gave rise to this impossibility, the *mutakallimūn* maintain, is only the doctrine of a potential infinity and an analysis of true potential. Thus, the objection concludes, the purported infinite divisibility associated with a continuum simply is not something truly potential, and so must be rejected along with the accompanying notion of a continuum.

As we have seen, however, infinite divisibility does not make up the essence of a continuum, according to Avicenna, but is only a description of it that must be demonstrated. Moreover, as we have also seen, for Avicenna the infinite divisibility of a continuum refers to the psychological process of positing limits within a continuous magnitude, and that process is in principle *never-ending*. Thus, *contra* the *mutakallimūn*, Avicenna can say that were this process ever completely actualized, then a process that in principle can go on without end must necessarily have come to an end, a blatant contradiction now on the part of the detractors of the continuous. Avicenna has safeguarded the possibility of continua from the attacks of the atomists.

4. AVICENNA ON THE *MINIMA NATURALIA*

Throughout his discussions of continuity and infinity, Avicenna keeps hinting at the possibility of various natural kinds having some physical limit of divisibility, which cannot be trespassed if the same natural substance is still to remain.⁵⁶ At *Physics* III.12, Avicenna finally takes up the issue of *minima naturalia* in earnest. Interestingly, the Latin translators of Avicenna's *Physics* stopped translating the *Physics* at III.10—no doubt in part, if not in full, because Avicenna provides some of his most thorough and trenchant arguments for the eternity of the cosmos there, a topic that the Church Schoolmen would have wanted to avoid.⁵⁷ Whatever the reasons for stopping at III.10, the end result is that neither medieval Latin philosophers nor modern historians of science have recognized or appreciated Avicenna's developed theory of *minima naturalia*.⁵⁸

⁵⁵See Wolfson, *The Philosophy of Kalām*, 469–70; and Dhanani, *The Physical Theory of Kalām*, 152–59.

⁵⁶See, for example, Avicenna, *Physics*, III.6, 4 and III.9, 4.

⁵⁷Since the Latin translators relegated chapter 1 to a prologue, the numbering of the Latin chapters is off by one when compared with the Arabic. Thus, the Arabic *Physics*, III.10 (the last chapter translated into Latin) corresponds with the Latin *Physics*, III.9.

⁵⁸A further effect was that neither medieval scholastics nor contemporary historians of science have fully appreciated Avicenna's theory of mixture, for it presupposes his theory of *minima naturalia*,

Avicenna begins *Physics* III.12, which treats not only *minima naturalia*, but also the related issues of whether there is a naturally smallest motion as well as a naturally fastest and a naturally slowest motion, with the following lines:

It is proper to add to these chapters [namely, those on continuous and atomic magnitudes] an investigation into the continuous preservation of the forms belonging to bodies and whether they retain [their forms] while being divided infinitely. In other words, just as bodies are infinitely divisible with respect to smallness and yet preserve the form of corporeality, do they likewise preserve the rest of the forms that they have—as, for example, [the substantial forms of] being water, air, and the like? (*Physics*, III.12, 1)

The issue of divisibility is raised again, but now Avicenna takes it as proven that a body as such is infinitely divisible with respect to its corporeality or being a body. This conclusion follows from his earlier discussion of continuity and division. One needs simply add the premise that for Avicenna being a body is nothing more than being a three-dimensional magnitude.⁵⁹ That is because to speak of magnitude just is to speak of the continuous. Moreover, as we saw, a continuous magnitude is infinitely divisible, since nothing prevents its being divided. Therefore, a body as such is likewise infinitely divisible. Such a conclusion is obviously the case for conceptual divisibility, but it equally holds for physical divisibility; for again there is nothing about the matter that would prevent the divisions, and while infinite divisibility is not the essence of a continuous magnitude, it still necessarily follows upon being a continuous magnitude.

The question Avicenna poses, then, is whether forms other than the form of corporeality can likewise weather such divisions. These other forms are the species forms—that is, the forms that make a given substance the natural kind that it is—and the “forms that belong to [bodies] on account of mixture (*mizāj*),”—as, for example, flesh, blood, and the like.⁶⁰ Avicenna immediately clarifies that the present discussion is not about the conceptual divisibility that is the product of the estimative faculty, although he does return to it again in the present chapter. Instead, the primary focus is about the divisions that seemingly reduce mixtures to their underlying constituents.⁶¹

The introduction of the forms of mixtures is significant because, recall, one of the other important places where later commentators found material relevant to *minima naturalia* is in Aristotle's discussion of mixture (*mixis/krama*). Avicenna is thus developing an integrated theory of *minima naturalia*, and in so doing he recognizes that it must also account for mixtures.⁶² Since the notion of mixture is a technical one in ancient and medieval science, a few words about it are warranted.⁶³

and the objections raised against it arise precisely from an ignorance of his theory of *minima naturalia*; see Maier, *An dGrenze von Scholastik und Naturwissenschaft*, 3–140, esp. 27–28.

⁵⁹Avicenna, *Kitāb al-Burhān*, I.10, 49; *Physics*, I.2, 2–4; and *Metaphysics*, II.2 (all).

⁶⁰Avicenna, *Physics*, III.12, 2.

⁶¹Avicenna, *Physics*, III.12, 2.

⁶²For a discussion of Avicenna's theory of mixture, see Stone, “Avicenna's Theory of Primary Mixture.”

⁶³The *locus classicus* is once again Aristotle's *On Generation and Corruption* I.10; other passages include *History of Animals* I.1, 487a2–10 and I.3, 511b1–10; *Parts of Animals* I.2, 640b18–29 and 2.2, 647b10–30; and *Generation of Animals* 2.6, 743a1–36. Equally important for understanding the ancient and medieval theory of mixture, particularly as it relates to medicine, is Galen's *On Mixtures* (*Peri Kraseōs*).

A mixture is best contrasted with generation/corruption and combination. In the case of generation and corruption, the initial ingredients become corrupted, that is, they completely cease to be and something totally new emerges. As such, the initial ingredients simply cannot be recovered. For example, once a loaf of bread has come to be, it is impossible to recover from the loaf the flour, water, salt, and yeast that were its initial ingredients. In contrast, in a mixture, while the initial ingredients are no longer actually present in the mixture, they are potentially there and so in principle can be recovered. Because in a mixture, the ingredients are no longer actually present but only potentially present, mixtures also differ from mere combinations, like in a blend of cornmeal and flour. In a combination the parts remain actually present and so the combination is not uniform throughout; cornmeal, for example, might be adjacent to flour. While much more can be said about the ancient and medieval theory of mixture, the previous two points suffice: one, the mixture must be uniform throughout; and, two, the ingredients of a mixture must be recoverable.⁶⁴

Returning to Avicenna's argument for *minima naturalia*, while he accepts their existence in both the cases of mixtures and the simple elemental bodies (like earth, air, etc.), he does not appear to be impressed with Aristotle's *Physics* 1.4 argument. Recall that Aristotle reasoned that if there were not *minima naturalia*, then the various natural kinds could be indefinitely small or large, but he also assumed that it was impossible for there to be indefinitely small and large natural kinds. Thus, Aristotle concluded that there must be *minima naturalia* in order to avoid this impossibility.

Although Avicenna does not say so explicitly, he seems to be uncomfortable with the gratuitous assumption that the existence of indefinitely small or large natural kinds is impossible. Remember that Philoponus too seemed to have reservations about this assumption, and even went so far as to suggest a proof for it. Again, Philoponus claimed that forms become attenuated when forced to spread throughout greater and greater magnitudes, and so there must be a natural maximum. He then went on simply to assert that, just as there must be a natural maximum, so must there be a natural minimum. Unfortunately, it is not clear why this conclusion follows, since in the case of minima, there is no longer the attenuation of forms, which was the key element in Philoponus's proof for natural maximums.

In Avicenna's first pass at Aristotle's argument, which in fact is a fairly free commentary on the *Physics* 1.4 passage, he attempts to reconstruct it without the troubling premise.

If this possibility concerning the generation of animals from their elements were a real one [that is, if one can extract out from the underlying mixture (*imtizāj*) indefinitely small amounts], not only would it be an absolute possibility, but also it would be a possibility that for the most part refers to what exists. That is because the mixture of the lesser part precedes the mixture of the greater part, for the greater part takes in the lesser part. The same holds for the account concerning composition [i.e. substances other than the elements]. It is more fitting that what is before should

⁶⁴For a detailed discussion of Aristotle's theory of mixture see Frede, "On Generation and Corruption I.10: On Mixture and Mixables."

exist than what is after, and so it is more fitting that mixtures from the smallest parts should exist. In that case, elephants the size of cats (to say nothing of the size of gnats!) would not have been so rare as to verge on the impossible. (*Physics*, III.12, 3)

Here we learn that the smaller mixture is prior to the larger mixture inasmuch as the greater mixture takes in the smaller, but not conversely. Presumably, Avicenna means something, for example, like this: While one can have a milliliter of some mixture without having a liter, one cannot have a liter of the mixture without having a milliliter. The milliliter goes into making up a liter in a way that the liter cannot go into making up the milliliter. He then continues that since it is more fitting that the prior exists than the posterior, and the smaller is prior to the greater in the sense explained, then if there are no natural minima, “unnaturally” small kinds should exist like cat-sized elephants or even gnat-sized elephants. Of course it is an empirical fact that we only ever experience various natural kinds within a certain definite range of sizes, and yet again preternaturally small instances of the various kinds should be the norm if there were not *minima naturalia*. Thus, Avicenna concludes that the mixtures making up the various natural kinds cannot be indefinitely small but have a natural minimum.

Avicenna's second reconstruction of Aristotle's argument focuses on the formal cause.⁶⁵ He claims in that argument that the mixture alone may not be sufficient for the occurrence of the species form and that a further condition may be necessary as well, namely that, the magnitude must be of sufficient size. The reason, he says, is that the proper function of the species form is to cause the various activities proper to the species, and it may turn out that these activities can only be performed if the body is of a certain size. Thus, he writes,

An example, then, is the fact that a human will be incapable of doing those things characteristic of a human unless his body is such as to perform human activities adequately. Not the least of [these activities] are that he have a power [*qūwa*] [that is, the form of human] and a tool [that is, a body] by means of which he can seek out and make a home (assuming there is no impediment), and by which he can fashion clothes, and [do] everything else a human must do to exist, as well as not being such that strong winds blow him about as so much dust and that the predominating lower qualities in him [namely, hot, cold, wet, and dry] do not change him. (*Physics*, III.12, 5)

Avicenna maintains that merely having the right kind of mixture, regardless of how small, is not enough to explain the occurrence of a given species form. The species form requires some natural minimum magnitude if that form is to produce the various species-appropriate activities identified with being a certain kind. As I noted earlier, Philoponus's attenuation argument for a natural maximum and minimum magnitude is better suited to a maximum magnitude than a minimum magnitude. The present argument seems ideally suited to make up for this weakness, since Avicenna's point is precisely that the power of a form becomes inefficacious if there is not a natural minimum magnitude in which that form subsists. This point becomes all the more germane in light of the fact that in places Avicenna identifies the power (*qūwa*) with the form itself.⁶⁶

⁶⁵Avicenna, *Physics*, III.12, 5.

⁶⁶See Avicenna, *On Generation and Corruption*, 6, 127.

Having offered his commentary and improvements on Aristotle's *Physics* 1.4 text, Avicenna turns to his own understanding of the *minima naturalia*, which relies heavily upon Philoponus's earlier comments as well as on Aristotle's *On Generation and Corruption* 1.10 and the commentary tradition surrounding that work.⁶⁷ He begins by reminding us that division is said in two ways: one that brings about an actual division, severance, or fragmentation of the magnitude, namely, physical division, and a second that involves a mere mental division or positing that leaves the magnitude intact, namely, conceptual division.

Concerning conceptual division, Avicenna is certain that there is no smallest conceptual part. Interestingly, instead of simply appealing to his earlier critique of atomism to make this point, he offers a different refutation of it, reminiscent of the "mathematicians' objection" noted by Philoponus, but now Avicenna turns that argument to his own advantage. On the strength of his earlier arguments, Avicenna takes as given that the form of a specific natural kind must "spread throughout the whole of [the body] with which it corresponds."⁶⁸ In other words, when one considers, for example, some mixture such as flesh, the form of flesh must spread throughout that mixture uniformly such that the flesh is a homogeneous homoeomery, that is, every part is the same throughout as the whole.

Since the present argument is about *conceptual* divisions, a brief comment on Avicenna's notion of conception (*taṣawwur*) is warranted. For Avicenna, *x* is conceivable, that is, one has a genuine concept of *x*, just in case there exists in one's intellect the corresponding intelligible for *x*, that is, the essence of *x* stripped of any material concomitants. A true intelligible, in contrast to some whim of the imagination or a vain intelligible, then, must correspond with some truly existing thing; otherwise, while one might be imagining *x*, one is not truly conceiving *x*. For example, while one might be able to form an image of a unicorn, that mere image is not the same as an intelligible concept of what a unicorn is—that is, of the essence of unicorn—and so the notion of a unicorn is a vain intelligible and so not truly conceivable, even if it is imaginable. This notion of conceivability underlies Avicenna's argument that there is no conceptual limit to the divisibility of homogeneous homoeomeries, such as elements and mixtures.

His argument is a *reductio*. Again, he takes the reality of homogeneous homoeomeries as given. Next, assume that there is a conceptual limit to their divisibility. In that case, if there were sufficiently miniscule magnitudes of which it is inconceivable that a given species form—whether an elemental form or the form of a mixture—can be found in that magnitude, then in those intervals either no body exists or a body of a different kind exists.

If some parts of the body were not to have their fair share of its form on account of their smallness, there would be an interval [*bu'd*] of parts alike in [that] status [that is, in not having that form] such that either the body would cease or parts smaller than them (and less likely to bear that form) would remain. In the [latter] case,

⁶⁷While it is obvious that Avicenna is responding to the commentary tradition surrounding *On Generation and Corruption*, only after a close comparison of Avicenna's own work with Philoponus's Greek commentary and the extant Arabic fragments of Alexander could one even begin to assess either the novelty of Avicenna's position or the degree of his indebtedness to the earlier tradition.

⁶⁸Avicenna, *Physics*, III.12, 6.

this body would be an ordered series of parts, none of which have this form. This form occurs only through the collection of [these parts], where the collection qua collection provides nothing but number and its properties, while qua a collection of bodies, it provides nothing beyond what the collection provides absolutely, save magnitude and its concomitants of shape and position. Now, none of that is fiery or earthy, not even in the parts taken separately nor in the whole of the collection. (Avicenna, *Physics*, III.12, 6)

The argument, which takes the form of destructive dilemma, is dense, and needs to be unpacked. The assumption to be rejected is that there are sufficiently small divisions that cannot even be conceived of sustaining, for example, the form of fire or flesh. In that case, assuming the conceptual division has been made, there is an interval in which either there is no form at all, or there is some form other than that of fire or flesh. The division is exhaustive, since the division leads either (1) to what has no form or (2) to what has some form, and the in-formed thing can be either (2a) something with the same kind of form or (2b) something with a different kind of form. Avicenna does not explicitly mention option (2a) because the assumption to be rejected by *reductio* is that one has reached a division where the existence of the initial form is no longer conceivable.

As for option (1), namely, that there is a formless interval, Avicenna dismisses it without comment, but his reason for doing so is easy enough to surmise. First, it is not clear how the whole substance would exist, if at the micro-level nothing is actual; for the form is the actuality of the substance, and yet it is being assumed that no form exists below a certain magnitude. In short, it was taken as given that a certain homogeneous homoeomery was actual, but it now seems that it is not actual, a contradiction. Second, and assuming that the present option allows some sense in which the whole substance exists, the language of 'interval' (*bu'd*), and particularly a formless interval, is Avicenna's preferred language for a void (*khalā'*). At *Physics* II.8, however, Avicenna had undertaken a detailed analysis and critique of the idea of void and found it empty, or more precisely, found it to be a vain intelligible. His criticism included interstitial voids too, and it may well be just such interstitial voids that the present option would allow.⁶⁹ Thus, this option leads to a known absurdity, namely, the existence of a void, and so there is no corresponding intelligible. As such, however, such a conceptual division in which there is a purported empty interval is inconceivable in the sense outlined above, and so the interval must be conceived to have some form

As for the second option, namely, that these conceptually small magnitudes are conceived to have a form other than that of fire and flesh, the larger magnitude of "fire" or "flesh" above this threshold would be nothing more than an ordered series or mere collection of parts, forming a heap but not a kind, like fire or flesh. In fact, on the current view, natural bodies would be no different from aggregates of atoms, none of which is flesh or fire. In effect, if there were limits on how small conceptual division could be, homoeomerous or uniform mixtures would be impossible, but the existence of homogeneous homoeomerics was taken as given and shown to be not only possible but also necessarily so in the early parts of book

⁶⁹It is perhaps also worth noting that Philoponus considered the idea of a "conceptual void" intelligible in a way that Avicenna did not; see McGinnis, "A Penetrating Question in the History of Ideas."

III of his *Physics*, and so there is a contradiction. Of course, it is impossible to have an intelligible that corresponds with a contradiction, and so again this horn of the dilemma is inconceivable given Avicenna's notion of conceivability. Conceptual divisibility *ad infinitum* is possible for Avicenna precisely because, upon reflection, the mind recoils from the idea that it cannot conceive of indefinitely small divisions of homogeneous homoeomeries.

As for physical divisibility, Avicenna contends that, unlike conceptual divisibility, there is in fact a natural minimum beyond which a given body is no longer able to preserve its species form. Avicenna now provides, as far as I can tell, a new proof for the necessity of *minima naturalia*. He argues thus:

Whenever bodies become smaller, they are increasingly disposed to being more quickly acted upon by other [bodies]. . . . So, apparently, when the body exceeds its degree of smallness and separates off from its collective kind, it would be impossible for it to retain its form at that time; but, rather, as a result of the bodies surrounding it, it will undergo alteration into them and become continuous with them. As such it will not maintain its form until mixed. (*Physics*, III.12, 8)

The argument is based upon, but not identical to, comments Aristotle makes at *On Generation and Corruption* 1.10, 328a18–35. Aristotle's text treats the possible reciprocal activity and passivity of substances in which it is mentioned in passing that the smaller the quantities the more easily they mix with one another. Additionally, in his commentary on this passage, Philoponus glosses Aristotle's remarks, saying, "forms are bound to a certain quantity in order to subsist, as was said in the *Physics*."⁷⁰ What is important to note is that while Philoponus appeals to *minima naturalia* (whose existence was purportedly proven in *Physics* 1.4) to explain Aristotle's comments, Avicenna turns Aristotle's comments into a new argument for *minima naturalia*.

Concerning the details of Avicenna's argument, 'collective kind,' as used in this passage, translates the Arabic '*kullīya*,' which, at least within the context of Avicenna's natural philosophy, is his preferred locution for the various elemental spheres of earth, water, air, and fire. Thus, in order to appreciate Avicenna's argument fully, I must briefly treat his understanding of the elements and substantial change, that is, change from being one kind of substance into being another kind of substance.

Associated with each of the traditional four elements are two primary qualities, one each from two sets of couplets. These couplets include the qualities hot/cold and wet/dry.⁷¹ Thus, the element earth is a predominately cold-dry mixture, water is a predominately cold-wet mixture, air is a predominately hot-wet mixture, and fire is a predominately hot-dry mixture. For Avicenna, these primary qualities are related to the elements' material cause. Now, when these primary qualities are altered—as, for example, when there is an increase or decrease in the degree of hotness, coldness, dryness, or wetness—the alteration prepares or predisposes a given element's matter so that it is suited to receive a new substantial form.⁷² So,

⁷⁰Philoponus, *In Phys.*, 198, 18–19.

⁷¹Avicenna, *On Generation and Corruption*, 9, 154–55.

⁷²The story is much more complex than I suggest here. It involves Avicenna's distinction between physical causation and metaphysical causation with its appeal to his celebrated Giver of Forms

for example, when water, which again is a cold-wet mixture, is sufficiently heated, the underlying qualitative disposition becomes no longer suited to the substantial form of water, and so at some point in the heating process the mixture receives a new substantial form that is compatible with the new underlying disposition, namely, it receives the form of air. And, indeed, the steam produced from vigorously heating water does have a definite air-like quality.

Bearing this account of the elements and substantial change in mind, we can now better appreciate Avicenna's proof for the existence of *minima naturalia*. The smaller the quantity of a given substance is, observes Avicenna, the more apt it is to be acted upon by surrounding bodies. This is the point that Avicenna draws from *On Generation and Corruption* 1.10. So, for example, all things being equal, it takes longer for a body of water to cool down a ton of molten iron than for that same body of water to cool down an ounce of molten iron, and similarly, a blast furnace is able to heat the same ounce more quickly than the ton. So, again, for Avicenna the smaller the physical divisions of a given substance are, the more disposed the resulting reduced quantity of the substance becomes to the primary qualities—again, hot, cold, wet, and dry—of the surrounding bodies. Below a certain limit, maintains Avicenna, the ratio between the strength of the primary qualities of the surrounding body and those of the body being divided is such that the qualities of the surrounding body overcome those of the divided body. At that moment, the divided substance's underlying mixture becomes no longer suitable for its elemental form, and the mixture receives a new substantial form.

So, for example, imagine a cup of water that is surrounded by hot summer air. Now, imagine half the amount of water, and then keep taking halves. At some point, the amount of water is so small that the water simply evaporates as it were instantaneously, or, as Avicenna would have it, the form of water in that minuscule physical quantity is immediately replaced with the form of air. In short, for Avicenna, the elements, as well as more complex mixtures like blood, flesh, and bone, have natural minima beyond which they cannot be divided and still be capable of retaining their species form; for the hot, cold, wet, and dry qualities of the surrounding bodies so alter that divided body that it is no longer suitable for sustaining its initial form. There must be, Avicenna thus concludes, *minima naturalia*. In this respect, Avicenna's *minima naturalia* are more like our modern notion of atoms than classical Greek and medieval Islamic accounts of atoms are. That is because if one takes a certain quantity of hydrogen, for example, and divides it, one is still left with hydrogen, that is, until one reaches a single hydrogen atom; for when a single hydrogen atom is divided, it is no longer hydrogen but a proton and an electron. The "form" of hydrogen is, as it were, lost. While not exact, at least in certain respects the analogy between *minima naturalia* and modern atoms is, then, apropos.

(*Wāhib al-ṣuwar*); see Janssens, "The Notions of «*Wāhib al-ṣuwar*» (Giver of Forms) and «*Wāhib al-'aql*» (Bestower of Intelligence) in Ibn Sīnā," and Hasse, "Avicenna's 'Giver of Forms' in Latin Philosophy, Especially in the Works of Albertus Magnus." Additionally, his account of substantial change requires an understanding of his critique of earlier theories of substantial forms, particularly his competing view of substantial form comes to play in his account of elemental change; see, Stone, "Avicenna's Theory of Primary Mixture," 109–19; and Richards, "Avicenna and Aquinas on Form and Generation."

I end this section with one final observation: Avicenna's theory of *minima naturalia* is a small mixture of *Physics* 1.4 and *On Generation and Corruption* 1.10. As such, Avicenna's theory of natural minima anticipates at many points his theory of mixture developed in his own *On Generation and Corruption*, 6. Two points in particular are salient. Recall that for Avicenna, when there are quantities of either the elements or mixtures that are equal to or greater than the naturally minimum quantity required for the substantial form, the form spreads uniformly throughout the substance such that the substance is a continuous whole. Inasmuch as there is a continuous whole, one can *conceptually* divide the substance as small as one likes, even below the naturally minimum quantity, and the substance remains uniform throughout. It is only when the substance is *physically* divided such that the parts are literally separated from one another that the mixture is destroyed and the ingredients of the mixture emerge once again. Thus, Avicenna's theory of *minima naturalia* with its reliance on two types of divisibility not only lets him account for the uniformity of mixtures, but also provides him with the mechanism for explaining recoverability, which again were two of Aristotle's desiderata for a true mixture.⁷³

In conclusion, until relatively recently the theory of a natural minimum was considered to be primarily the product of Latin Schoolmen working from a number of germinal statements in Aristotle. Recently, Ruth Glasner and others have shown that, despite the limited knowledge that medieval Churchmen and historians of science have had concerning Averroes's theory of *minima naturalia*, he in fact had a very developed account in which he grappled with both interpretative problems within Aristotle and substantive philosophical issues. Hopefully, I have shown that Averroes was not the only philosopher in the Arabic world working on this issue, and that Avicenna was at least as vigorously engaged in the issue of *minima naturalia*. Moreover, I strongly suspect that Avicenna's work may have strongly influenced Averroes's own reading of Aristotle. Finally, I doubt that Avicenna and Averroes were the sole thinkers within the medieval Arabic tradition of Neoplatonized Aristotelian philosophy to undertake this endeavor. Thus, what still waits to be written is the full story of *falsafa's* contribution to the history of atomism.

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⁷³While more can be said about the relation between Avicenna's theories of *minima naturalia* and mixture, I believe that the foregoing offers a corrective to Abraham Stone's otherwise excellent analysis of Avicenna's mixture; for he claims that for Avicenna "small elemental bodies are still present in the complex [i.e. mixture]," and so, according to Stone, they are in a real sense *actually* present in the mixture ("Avicenna's theory of Primary Mixture," 113). Against the backdrop of Avicenna's theory of *minima naturalia*, what one sees is that while the elemental bodies can be recovered on Avicenna's account, they only are potentially in the mixture, not actually there, and even 'potentially' must be understood against the background of Avicenna's theory of continuity.

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